MATH-821 Analytical Approximate Solutions of ODEs

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The objective of this course is to introduce analytical and approximate methodsfor differential equations and make students familiar with advanced topics in spectral methods.

Core Contents: The variational iteration method, The Adomian decomposition method, Perturbation method, Hamiltonian approach, Homotopy analysis method, spectral methods, Fourier and Chebyshev Series, Discrete least square approximation, Chebyshev interpolation, Tau-spectral method. Collocation spectral methods.

Detailed Course Contents: The variational iteration method: Application of the variational iteration method. The Adomian decomposition method: Application of the Adomian decomposition method. Perturbation method: Theoretical background, application of the perturbation method. Energy balance method: Theoretical background, application of the energy balance method. Hamiltonian approach: Theoretical background, application of the Hamiltonian approach. Homotopy analysis method: Theoretical background. Homotopy analysis method: application of the homotopy analysis method.
Fourier and Chebyshev Series, The trigonometric Fourier series. The Chebyshev series.Discrete least square approximation. Chebyshev discrete least square approximation. Orthogonal polynomials least square approximation. Orthogonal polynomials and Gauss-type quadrature formulas. Chebyshev projection. Chebyshev interpolation. Collocation derivative operator. General formulation for linear problems. Tau-spectral method. Collocation spectral methods: A class of nonlinear boundary value problems. Spectral-Galerkinmethods.

Learning Outcomes: On successful completion of this course students will be able to:

- Understand and apply approximate methods such as the variational iteration method,
- The Adomian decomposition method, Perturbation method, Hamiltonian approach,Homotopy analysis method
- Understand and apply spectral methods for solving differential equations.

Textbooks:

1. M. Hermann, M. Saravi, (HS) Nonlinear Ordinary Differential Equations, AnalyticalApproximations and Numerical Methods, Springer (2016)
2. C. I. Gheorghiu, (CIG) Spectral Methods for Differential Problems, TiberiuPopoviciuInstitute of Numerical Analysis (2007)

Reference Book:

1. C. Canuto, M. Y. Hussaini, A. Quarteroni and T. A. Zang, Spectral Methods: Fundamentals inSingle Domains, Springer (2006)
2. Lloyd N. Trefethen, Approximation Theory and Approximation Practice, Siam (2013).

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	***Section***	***Topics***
1	HS 2.1-2.3	The variational iteration method, application of the variational iteration method.
2	2.4, 2.5	The Adomian decomposition method, application of the Adomian decomposition method.
3	3.1	Perturbation method: theoretical background, application of perturbation method.
4	3.2	Energy balance method: theoretical background, application of energy balance method.
5	3.3	Hamiltonian approach: theoretical background, application of the Hamiltonian approach.
6	3.4	Homotopy analysis method: theoretical background.
7	3.4 (cont.)	Homotopy analysis method: application of the homotopy analysis method.
8	1.1,1.2.1,1.2.2	General properties, Fourier and Chebyshev Series, The trigonometric Fourier series, The Chebyshev series.
9	**Mid Semester Exam**	
10	1.2.3	Discrete least square approximation.
11	1.2.4,1.2.6	Chebyshev discrete least square approximation, Orthogonal polynomials least square approximation, Orthogonal polynomials and Gauss-type quadrature formulas
12	1.3,1.4	Chebyshev projection, Chebyshev interpolation.
13	1.4 (cont.)2.1	Chebyshev interpolation (cont.) Collocation derivative operator. The idea behind the spectral methods.
14	2.2,2.3	General formulation for linear problems, Tau-spectral method.
15	2.4	Collocation spectral methods (pseudo spectral), A class of nonlinear boundary value problems.
16	2.5	Spectral-Galerkin methods.
17		Review
18	**End Semester Exam**	